Published by Institute of Physics Publishing for SISSA
Received: *January 6, 2005*
Revised: *February 16, 2005*
Accepted: *February 24, 2005*
Published: *April 18, 2005*

Strings in a 2-d extremal black hole

Amit Giveon and Amit Sever

Racah Institute of Physics, The Hebrew University
Jerusalem 91904, Israel
E-mail: giveon@vms.huji.ac.il, asever@phys.huji.ac.il

Abstract: String theory on 2-d charged black holes corresponding to $\frac{\mathrm{SL}(2)\times\mathrm{U}(1)_L}{\mathrm{U}(1)}$ exact asymmetric quotient CFTs are investigated. These backgrounds can be embedded, in particular, in a two dimensional heterotic string. In the extremal case, the quotient CFT description captures the near horizon physics, and is equivalent to strings in AdS_2 with a gauge field. Such string vacua possess an infinite space-time Virasoro symmetry, and hence enhancement of global space-time Lie symmetries to affine symmetries, in agreement with the conjectured AdS_2/CFT_1 correspondence. We argue that the entropy of these 2-d black holes in string theory is compatible with semi-classical results, and show that in perturbative computations part of an incoming flux is absorbed by the black hole. Moreover, on the way we find evidence that the 2-d heterotic string is closely related to the $N = (2, 1)$ string, and conjecture that they are dual [3].

Keywords: Superstrings and Heterotic Strings, Conformal Field Models in String Theory, Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

In this note we consider string theory on a two dimensional Extremal Black Hole (EBH), corresponding to the exact maximally asymmetric quotient Conformal Field Theory (CFT) background $\frac{\mathrm{SL}(2,\mathbb{R})\times\mathrm{U}(1)_L}{\mathrm{U}(1)}$. Strings propagating in 2-d black holes may serve as useful toy models for the study of black holes in a theory of quantum gravity (see [4] for a review). Our interest in this particular 2-d EBH is four fold:

1. It is an exact CFT background with a *constant* dilaton, which thus can be used to study some aspects of the black hole physics in standard perturbative string theory.

2. It has an infinite space-time Virasoro symmetry which may allow a better understanding of its properties, such as the entropy.

3. It can be embedded in a *two dimensional* heterotic string theory, which might have a useful Conformal Quantum Mechanics dual.

4. It is equivalent to string theory in AdS_2 with a gauge field, and hence might shed more light on the conjectured AdS_2/CFT_1 correspondence or, alternatively, be analyzed non-perturbatively by using this duality.

We begin in section 2 by reviewing the 2-d Charged Black Holes corresponding to a family of exact CFTs — the (left-right asymmetric) quotients $\frac{\mathrm{SL}(2,\mathbb{R})\times \mathrm{U}(1)}{\mathrm{U}(1)}$ [5]–[9]. In section 3 we consider the extremal case, and show that its corresponding exact CFT — the maximally asymmetric quotient — captures the near horizon physics, and is equivalent to an AdS_2 background with a gauge field. In section 4 we sketch the construction of various string vacua on EBH $\times \mathcal{N}$, and in section 5 we construct a two dimensional heterotic string on EBH $\equiv AdS_2$. We show that in all string vacua on EBH there is a space-time Virasoro symmetry, similar to string theory on AdS_3 [10], which enhance some global Lie symmetries to affine symmetries in the conjectured space-time dual Quantum Mechanics. On the way, we find evidence that the 2-d heterotic string is closely related to the $N = (2, 1)$ string [3]. In section 6 we discuss the entropy of these 2-d black holes, evaluated by using some aspects of its non-perturbative dual, and recall that the reflection coefficient for waves scattered from its event horizon, evaluated perturbatively in its exact CFT description, is smaller than one. Finally, in appendix A we describe AdS_2 in various coordinate systems, and in appendix B we show that the EBH background is equivalent to a maximally asymmetric orbifold of AdS_3 — the extremal BTZ black hole.

2. 2-d charged black holes as exact CFTs — a review

We consider string theory on the background CBH $\times \mathcal{N}$, where CBH is the 2-d Charged Black Hole background (described below) and $\mathcal{N}$ is a compact Conformal Field Theory background. The CBH background is the exact $\frac{\mathrm{SL}(2,\mathbb{R})_k\times \mathrm{U}(1)}{\mathrm{U}(1)}$ quotient CFT background obtained by the anomaly free asymmetric gauging

$$(g, x_L, x_R) \simeq \left(e^{\tau\cos(\psi)\sigma_3/\sqrt{k}} g e^{\tau\sigma_3/\sqrt{k}}, x_L + \tau\sin(\psi), x_R\right). \tag{2.1}$$

Here $g \in \mathrm{SL}(2,\mathbb{R})_k$ — an $\mathrm{SL}(2,\mathbb{R})$ WZW model at level k – $(x_L, x_R) \in \mathrm{U}(1)_L \times \mathrm{U}(1)_R$, where L, R stand for left- and right-movers, respectively. Note that the gauging (2.1) does not act on x_R and, therefore, this background can be embedded, in particular, in a 2-d heterotic string, where x_R does not exist and with x_L being part of its chiral internal space (see section 5).

The sigma-model background corresponding to this exact $\frac{\mathrm{SL}(2,\mathbb{R})_k\times \mathrm{U}(1)}{\mathrm{U}(1)}$ CFT is three dimensional. Taking a small U(1) radius (relative to the $\mathrm{SL}(2,\mathbb{R})$ radius $\sqrt{k}$), a Kaluza-Klein (KK) reduction to two dimensions with a gauge field is justified. In a 2-d heterotic string this background is $\frac{\mathrm{SL}(2,\mathbb{R})_k\times \mathrm{U}(1)_L}{\mathrm{U}(1)}$, which is two dimensional with a gauge field.

To obtain the sigma-model background, we first choose to parametrize

$$g(z, y, \theta; \delta) = e^{(z+y)\sigma_3/2} g_\delta(\theta) e^{(z-y)\sigma_3/2}. \tag{2.2}$$

For each choice of $g_\delta(\theta)$ this parametrization covers a certain region of the $\mathrm{SL}(2,\mathbb{R})$ group manifold (or its universal cover – $\tilde{S}L(2,\mathbb{R}) \equiv AdS_3$). We denote by regions A, B, C the ones corresponding to

$$g_A(\theta) = e^{\theta\sigma_1}\,, \qquad g_B(\theta) = e^{i\theta\sigma_2}\,, \qquad g_C(\theta) = e^{\theta\sigma_1} i\sigma_2\,, \tag{2.3}$$

respectively. After gauging (2.1) (with the gauge fixing choice $z = 0$) and a KK reduction one obtains, in each region, a two dimensional metric, a dilaton Φ and a gauge field A_y (for details see [8, 9]):[1]

$$\begin{aligned}
A: \quad \frac{1}{k}ds^2 &= d\theta^2 - \frac{\coth^2(\theta)}{(\coth^2(\theta) - p^2)^2}dy^2\,, \\
\frac{1}{\sqrt{k}}A_y &= \frac{p}{p^2 - \coth^2(\theta)}\,, \qquad \theta \geq 0\,, \\
\Phi &= \Phi_0 - \frac{1}{2}\log\left(\cosh^2(\theta) - p^2\sinh^2(\theta)\right)\,,
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
B: \quad \frac{1}{k}ds^2 &= -d\theta^2 + \frac{\cot^2(\theta)}{(\cot^2(\theta) + p^2)^2}dy^2\,, \\
\frac{1}{\sqrt{k}}A_y &= \frac{p}{p^2 + \cot^2(\theta)}\,, \quad 0 \leq \theta \leq \frac{\pi}{2}\,, \\
\Phi &= \Phi_0 - \frac{1}{2}\log\left(\cos^2(\theta) + p^2\sin^2(\theta)\right)\,,
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
C: \quad \frac{1}{k}ds^2 &= d\theta^2 - \frac{\tanh^2(\theta)}{(\tanh^2(\theta) - p^2)^2}dy^2\,, \\
\frac{1}{\sqrt{k}}A_y &= \frac{p}{p^2 - \tanh^2(\theta)}\,, \quad \theta \geq 0\,, \\
\Phi &= \Phi_0 - \frac{1}{2}\log\left(\sinh^2(\theta) - p^2\cosh^2(\theta)\right)\,.
\end{aligned} \tag{2.6}$$

Here,

$$p^2 = \tan^2\left(\frac{\psi}{2}\right)\,, \qquad \psi \in \left[0, \frac{\pi}{2}\right]\,, \tag{2.7}$$

where w.l.g. we chose $p^2 \leq 1$, and Φ_0 is a constant, related to the dilaton on the group $SL(2,\mathbb{R})$ theory by

$$\Phi_{sl(2)} = \Phi_0 - \frac{1}{2}\log\left(\frac{1+p^2}{2}\right)\,. \tag{2.8}$$

Region A is obtained from region B by taking $\theta \to i\theta$, while region C is obtained from B by $\theta \to i\theta + \pi/2$ [8].

Next we do a coordinate transformation

$$t = \frac{2y}{1-p^2}\,, \qquad r = \frac{2e^{-2\Phi}}{\sqrt{k}}\,. \tag{2.9}$$

In these Schwarzschild-like coordinates, the background (2.4)–(2.6) takes the form

$$\frac{4}{k}ds^2 = -f(r)dt^2 + \frac{dr^2}{r^2 f(r)}\,, \qquad \frac{1}{\sqrt{k}}A_t = \frac{Q}{2}\left(\frac{1}{r_+} - \frac{1}{r}\right)\,, \tag{2.10}$$

[1] We set $l_s = 1$.

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}\,, \tag{2.11}$$

with

$$\sqrt{k}M = (1+p^2)e^{-2\Phi_0}\,, \qquad \sqrt{k}Q = 2pe^{-2\Phi_0}\,, \tag{2.12}$$

and

$$r_\pm = M \pm \sqrt{M^2 - Q^2}\,. \tag{2.13}$$

Note that

$$p^2 = \frac{r_-}{r_+}\,, \tag{2.14}$$

where p^2 is defined in eq. (2.7).

This background is a 2-d CBH with mass M and charge Q — the one discovered and studied in [5]. The inner (Cauchy) and outer (event) horizons are located at $r = r_\pm$, and the singularity is located at $r = 0$. Region A (2.4) is mapped to the region outside the event horizon $r_+ < r < \infty$. Region B (2.5) is mapped to the region between the outer and inner horizons $r_- < r < r_+$. Region C is mapped to the region beyond the inner horizon $r < r_-$; the region beyond the singularity corresponds to $r < 0$ or, equivalently, a positive r but $M < 0$.[2] The maximal extension of the CBH is given by taking the infinite regions obtained from gauging the universal cover of $\mathrm{SL}(2,\mathbb{R})$; see [8, 9] for details.

In type-II and type 0 string theory on CBH$\times\mathcal{N}$ the CBH part is a SCFT on $\frac{\mathrm{SL}(2,\mathbb{R})\times\mathrm{U}(1)}{\mathrm{U}(1)}$. The central charge of the CBH superconformal sigma model is $c_{cbh} = (3 + 6/k) + 3/2$ and $c_\mathcal{N} = 15 - c_{cbh}$, both for the left- and right-movers. In the heterotic string, for which x_R does not exist, $c_{cbh} = 3 + 6/k$ and $c_\mathcal{N} = 15 - c_{cbh}$ for the fermionic right-moving sector, while $c_\mathcal{N} = 26 - c_{cbh}$ for the bosonic left-moving sector once we embed the SCFT on CBH in the bosonic sector and include the $\mathrm{U}(1)_L$ connected to the EBH gauge field in $\mathcal{N}_L$. Finally, in the bosonic string $c_{cbh} = 3 + 6/k$, with $k = k_{\mathrm{bosonic}} - 2$, and $c_\mathcal{N} = 26 - c_{cbh}$ both for the left- and right-movers.

In string theory, generically the exact coset CFT backgrounds corresponding to the CBH receive higher order $\alpha' \sim 1/k$ corrections (see e.g. [13, 14] and references therein). In string vacua corresponding to "left-right symmetric" supersymmetric quotients we do not expect such corrections to the metric [15, 16, 17, 18, 13, 14].[3]

3. 2-d extremal black holes as exact CFTs

We now consider string theory on the background EBH$\times\mathcal{N}$, where EBH is the 2-d Extremal Black Hole. This is, apparently, a special case of the discussion in the previous section for which $M^2 = Q^2$ or, equivalently, $r_+ = r_-$ ($p^2 = 1$). In this case $\psi = \pi/2$ (2.7) and hence the gauging (2.1) acts only on $\mathrm{SL}(2,\mathbb{R})_R \times \mathrm{U}(1)_L$:

$$(g, x_L, x_R) \simeq \left(ge^{\tau\sigma_3/\sqrt{k}}, x_L + \tau, x_R\right). \tag{3.1}$$

[2] In string theory, one may consider also the region beyond the singularity [11, 4, 8, 12].

[3] For the 2-d heterotic string in such quotients – where the background is supplemented by a (higher order in α') gauge field which is set equal to the Lorentz connection – the metric does not receive α' corrections; see [14] for details.

In the “group coordinates” y, θ or, for convenience,

$$\chi = 2\theta \,, \tag{3.2}$$

we find the following metric and gauge field in the various regions. The 2-d metric in regions A and C (2.4), (2.6) is now identical, while the gauge field is the same up to a sign (which can be changed by $y \to -y$) and an additive constant,

$$A, C: \quad \frac{4}{k} ds^2 = d\chi^2 - \sinh^2(\chi) dy^2 \,, \qquad \frac{1}{\sqrt{k}} A_y = \frac{1}{2}(1 \mp \cosh(\chi)) \,, \tag{3.3}$$

where the $(\mp)$ in the gauge field is $(-)$ in region A and $(+)$ in C. In region B (2.5), we have

$$B: \quad \frac{4}{k} ds^2 = -d\chi^2 + \sin^2(\chi) dy^2 \,, \qquad \frac{1}{\sqrt{k}} A_y = \frac{1}{2}(1 - \cos(\chi)) \,. \tag{3.4}$$

The dilaton is constant, and equals in all the regions (2.4), (2.5), (2.6) to its value in the group (2.8):

$$\Phi = \Phi_0 = \Phi_{sl(2)} \,. \tag{3.5}$$

This background is the same one obtained in [2, 19] by a certain compactification of AdS_3 on a circle, and it is equivalent to a KK reduction of the extremal BTZ black hole to two dimensions; see appendix B for details. This background and its properties are also obtained by a certain current-current deformation of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$ in [20], along the lines of [21].

Each of the backgrounds (3.3) and (3.4) covers a different patch of AdS_2 (see appendix A for more details on AdS_2). To show this, we first represent AdS_2 (with a unit radius, for simplicity) as the infinite cover of the hyperboloid

$$X_0^2 + X_2^2 - X_1^2 = 1 \,, \tag{3.6}$$

in $\mathbb{R}^{2,1}$ with metric

$$ds^2 = -dX_0^2 - dX_2^2 + dX_1^2 \,. \tag{3.7}$$

Parametrizing

$$\begin{aligned} X_0 &= \sinh(\chi) \sinh(y) \,, \\ X_1 &= \sinh(\chi) \cosh(y) \,, \\ X_2 &= \cosh(\chi) \,, \end{aligned} \tag{3.8}$$

one obtains the metric (3.3). Hence, region A (or C) covers a patch of AdS_2 with $X_2 > 1$ and $X_1 > 0$ (or $X_2 < -1$ and $X_1 < 0$[4]). Similarly, parametrizing

$$\begin{aligned} X_0 &= \sin(\chi) \cosh(y) \,, \\ X_1 &= \sin(\chi) \sinh(y) \,, \\ X_2 &= \cos(\chi) \,, \end{aligned} \tag{3.9}$$

one obtains the metric (3.4). Hence, region B covers the patch $-1 < X_2 < 1$ and $X_0 > 0$ of AdS_2.

[4]Recall that region C is obtained from A by $\chi \to \chi - i\pi$ (see eq. (26) in [8]).

JHEP02(2005)065

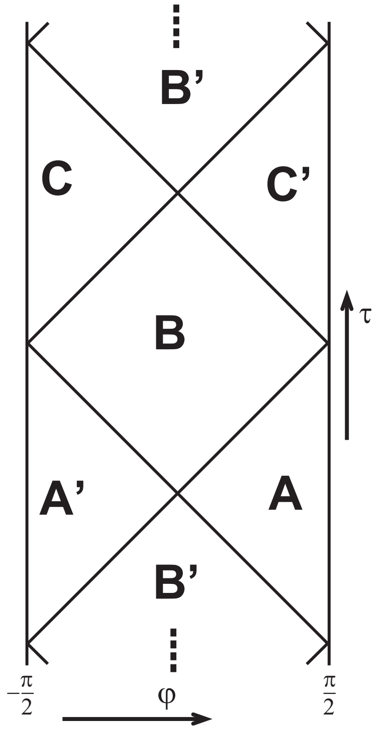

Figure 1: The regions of the Extremal Black Hole in AdS_2; τ, φ are the coordinates of section A.2 and $A, B, C \leftrightarrow A', B', C'$ under $\chi \leftrightarrow \chi'$ in (3.8), (3.9).

To get the maximal extension of the EBH, one considers the infinite regions obtained from gauging the universal cover of $\mathrm{SL}(2, \mathbb{R})$ (see [8, 9] for details). This maximally extended EBH covers the full AdS_2 – the universal cover of the hyperboloid (3.6) (see appendix A). The location of the various regions in the AdS_2 strip is described in figure 1.

What happens if instead we take $M^2 = Q^2$ in the 2-d background written in the Schwarzschild-like coordinates (2.10), (2.11)? As in the non-extremal case, one finds an asymptotically flat background with a linear dilaton, namely, at $r \to \infty$:

$$ds^2 \to \frac{k}{4}(-dt^2 + d\phi^2)\,, \qquad \Phi \to -\frac{1}{2}\phi + \text{const.}\,, \qquad A_t \to \text{const.}, \qquad r = e^{\phi}\,. \tag{3.10}$$

This background is thus different from the AdS_2 background above. The reason for this is that the coordinate transformation (2.9) is ill defined in the extremal case: the dilaton is a constant (3.5) and hence r in eq. (2.9) is fixed at the horizon (2.13), (2.12): $r = M$. Consequently, to compare to the quotient CFT background in the $p = 1$ case, we are led to investigate the near horizon limit of the extremal 2-d black hole.

The near horizon limit is obtained, for instance, by inserting $r = M + u$ in (2.10) with $f(r) = (1 - M/r)^2$, and keeping only terms with $u \ll M$. After rescaling t, one obtains

$$\frac{4}{k}ds^2 = -u^2dt^2 + \frac{du^2}{u^2}\,, \qquad \frac{1}{\sqrt{k}}A_t = \frac{u}{2}\,. \tag{3.11}$$

For $u \geq 0$, this background is the Poincaré patch of AdS_2 — it covers one half of the hyperboloid (3.6) (see appendix A and [22] for a review) — with a gauge field.

We have thus learned that the exact CFT background $\frac{\mathrm{SL}(2,\mathbb{R})\times \mathrm{U}(1)}{\mathrm{U}(1)}$ with the gauged U(1) acting in a maximally asymmetric way (3.1) describes both the physics of string theory in the near horizon limit of an extremal 2-d black hole or, equivalently, in an AdS_2 background with a gauge field. Another relation between (3.3), (3.4) and an AdS_2 black hole is described in subsection A.4 of appendix A.

4. String theory on EBH $\times \mathcal{N}$

We now discuss various perturbative string vacua (see [23, 24] for a review) in the EBH background. In subsection 4.1 we begin with the bosonic string, in subsection 4.2 we sketch the construction of type-II and type 0 fermionic string vacua, and in subsection 4.3 the heterotic string is introduced.

4.1 Bosonic string theory on EBH $\times \mathcal{N}$

The bosonic string theory has tachyons, but as usual it is constructive to consider first the bosonic string on EBH $\times \mathcal{N}$. In this case EBH $\times \mathcal{N}$ is a CFT background with $c = 26$. Physical vertex operators are obtained by dressing an operator $V_{\mathcal{N}}$ in the CFT $\mathcal{N}$ with an operator in EBH, such that the on-shell mass condition and other physical conditions are satisfied. For instance, some vertex operators in EBH are obtained from primaries of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$,

$$V_{jm\bar{m}} e^{i(k_L x_L + k_R x_R)}\,, \tag{4.1}$$

by imposing the gauge condition (see eq. (52) in [8])

$$\bar{m} = \sqrt{k} k_L\,. \tag{4.2}$$

Here $V_{jm\bar{m}}$ is a primary of $\mathrm{SL}(2,\mathbb{R})_L \times \mathrm{SL}(2,\mathbb{R})_R$ with Casimir $-j(j+1)$ and $(J^3, \bar{J}^3)$ eigenvalues equal to $(m, \bar{m})$,[5] and (k_L, k_R) are momenta on the even self-dual Narain lattice $\Gamma^{1,1}$. In particular, for uncharged states in EBH ($k_L = 0$) the gauge condition (4.2) reads $\bar{m} = 0$. Thus, while the right-moving $\mathrm{SL}(2,\mathbb{R})_R$ symmetry is broken to U(1) (only the generator $\bar{L}_0$ survives the gauge condition), the left-moving $\mathrm{SL}(2,\mathbb{R})_L$ survives (4.2). Moreover, the global $\mathrm{SL}(2,\mathbb{R})$ Lie algebra – generated by $L_0, L_{\pm 1}$ – is enhanced to a full Virasoro symmetry in target space, generated by the operators L_n with $n \in \mathbf{Z}$, constructed in eq. (2.36) of [10].

The AdS_2 nature of EBH suggests that this Virasoro symmetry is the conformal group of a certain Conformal Quantum Mechanics, as expected in the AdS_2/CFT_1 conjecture [26] (see [22] for a review). For a closely related discussion see [19].

[5] We actually consider the CFT on euclidean AdS_3, for which $(m, \bar{m})$ in eq. (4.1) are real. To obtain results in the lorentzian black hole we have to analytically continue $m \to im$ (see [8, 25] for more details).

4.2 Type-II and 0 string theory on $\mathrm{EBH} \times \mathcal{N}$

To construct fermionic strings we consider the SCFT on $\mathrm{EBH} \times \mathcal{N}$ with a central charge $c = 15$. If the CFT background $\mathcal{N}$ has an affine $\mathrm{U}(1)$ symmetry, and $\mathcal{N}/\mathrm{U}(1)$ has an $N = 2$ supersymmetry, the worldsheet theory on $\mathrm{EBH} \times \mathcal{N}$ allows to construct string vacua with space-time superconformal symmetries. Such superstrings on $\mathrm{EBH} \times \mathcal{N}$ can be constructed in a similar way to the supersymmetric string vacua on $AdS_3 \times \mathcal{N}$, studied in [27, 28, 29]. Generically, only one half of the superconformal symmetries in AdS_3 survive in the superstring on EBH – the ones associated with the $\mathrm{SL}(2,\mathbb{R})_L$ left-moving symmetry, which survives the gauge condition (4.2). Type 0 string theory on $\mathrm{EBH} \times \mathcal{N}$ can be obtained in a standard way. Both type-II and 0 string vacua on EBH have a space-time Virasoro symmetry generated by the operators L_n constructed in eq. (3.5) of [10]. The construction of physical vertex operators in these theories is a straightforward generalization of the discussion in $AdS_3 \times \mathcal{N}$ [10, 30, 31] and the previous subsection, with the gauge condition (4.2), the appropriate mass shell condition, other physical conditions and GSO projection applied.

4.3 Heterotic string theory on $\mathrm{EBH} \times \mathcal{N}$

In the heterotic string on $\mathrm{EBH} \times \mathcal{N}$ we consider the superconformal extension of of EBH, and embed it such that the right-moving SCFT is part of the fermionic sector, while the left-moving SCFT is part of the bosonic sector. Moreover, w.l.g. we choose x_L in (3.1) to be part of the chiral internal space of $\mathcal{N}$, hence x_R does not exist. Consequently, the sigma model background obtained by the gauging (3.1) is a *two dimensional* AdS_2 background with a gauge field, given in (3.3), (3.4).

As in the bosonic string, the left-moving Virasoro generators L_n survive the gauge condition (4.2). Hence the target space dual — via the conjectured AdS_2/CFT_1 correspondence — is expected to be a certain Conformal Quantum Mechanics with a full Virasoro algebra as its conformal symmetry.

A particularly interesting case is the two dimensional heterotic string on EBH or, equivalently, the heterotic string on AdS_2. This is the topic of the next section.

5. 2-d heterotic string theory on $\mathrm{EBH} \equiv AdS_2$

The structure of the CBH background — $\frac{\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)_L}{\mathrm{U}(1)}$ – allows us to embed it in a two dimensional heterotic string theory. To discuss the 2-d heterotic string on EBH (or, equivalently, on AdS_2 with a gauge field), we first consider the slightly simpler, closely related case of the $(2,2)$ heterotic string on $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ – the uncharged 2-d black hole.

The right-moving fermionic sector of this heterotic string consists entirely of the right-moving sector of the SCFT on $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$, hence the $\mathrm{SL}(2,\mathbb{R})$ level is $k = 1/2$, such that $c_{\mathrm{fermionic}} = 3 + 6/k = 15$ is critical. The SCFT on $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ has an $N = (2,2)$ supersymmetry. The right-moving $\mathrm{U}(1)_{\mathcal{R}}$ current of this $N = 2$ algebra is $\bar{J}_{\mathcal{R}} = i\sqrt{5}\bar{\partial}\bar{H}$, where $\bar{H}(\bar{z})$ is a canonically normalized scalar: $\bar{H}(\bar{z})\bar{H}(\bar{w}) \sim -\log(\bar{z} - \bar{w})$. Space-time supercurrents correspond to chiral spin fields on the worldsheet:

$$Q_\pm \equiv \int d\bar{z} e^{-\frac{\bar{\varphi}}{2}} e^{\pm\frac{i}{2}\sqrt{5}\bar{H}}\,, \tag{5.1}$$

where $\bar{\varphi}(\bar{z})$ is the scalar appearing in the bosonization of the superconformal ghosts.

The left-moving bosonic sector consists of the same $N = 2$ SCFT with central charge $c = 15$, and with a U(1) $\mathcal{R}$-current:

$$J_{\mathcal{R}} = i\sqrt{\frac{c}{3}}\partial H = i\sqrt{5}\partial H\,, \tag{5.2}$$

where $H(z)$ is a canonically normalized scalar. In addition, there are 22 free chiral fermions $\lambda^A(z)$, $A = 1, \ldots, 22$, such that the total central charge is critical: $c_{\text{bosonic}} = 15 + \frac{1}{2} \times 22 = 26$.

One can obtain consistent, stable 2-d heterotic string vacua by either performing a diagonal GSO projection or a chiral GSO projection – mutual locality with (5.1).[6] Moreover, there are various ways to construct such modular invariant heterotic vacua, each is specified by its gauge symmetry. The latter is generically generated by left-moving currents. We now discuss two consistent heterotic vacua — those which correspond to a symmetric embedding of the gauge connection in the spin connection:

1. $\mathrm{U}(1) \times \mathrm{SO}(22)$: We can construct a consistent heterotic string background with $\frac{22\times 21}{2}$ holomorphic currents in the adjoint representation of SO(22):

   $$\lambda^A\lambda^B\,, \qquad A, B = 1, \ldots, 22 \qquad \in \text{ Adjoint of } \mathrm{SO}(22)\,. \tag{5.3}$$

   Together with the $\mathrm{U}(1)_{\mathcal{R}}$ current $i\partial H$ in (5.2) we have a $\mathrm{U}(1) \times \mathrm{SO}(22)$ gauge symmetry.

2. $\mathrm{SU}(5) \times E_8$: This heterotic string has an E_8 gauge symmetry generated by chiral currents in the adjoint of SO(16), given by 120 bilinears in 16 of the λ's, say $\lambda^7, .., \lambda^{22}$, and the spinor 128 of this SO(16) generated by the spin fields of these 16 λ's. In addition there is an $E_4 \equiv \mathrm{SU}(5)$ gauge symmetry, whose holomorphic currents are:

   $$i\partial H\,, \qquad \lambda^a\lambda^b, a, b = 1, \ldots, 6\,, \qquad e^{\frac{i}{2}\sqrt{5}H}S_\alpha\,, \qquad e^{-\frac{i}{2}\sqrt{5}H}S_{\bar{\alpha}}\,, \tag{5.4}$$

   where S_α and $S_{\bar{\alpha}}$ are spin fields in the 4 and $\bar{4}$ representations of SO(6), respectively. Hence, the currents (5.4) are in the adjoint $(1 \oplus \frac{6\times 5}{2})$ plus spinor and spinor bar $(4 \oplus \bar{4})$ of $\mathrm{U}(1) \times \mathrm{SO}(6)$, respectively, which is equivalent to the adjoint of SU(5):

   $$1 \oplus 15 \oplus 4 \oplus \bar{4} \text{ of } \mathrm{U}(1) \times \mathrm{SO}(6) = 24 \text{ of } \mathrm{SU}(5)\,. \tag{5.5}$$

   We thus find that the gauge symmetry is $\mathrm{SU}(5) \times E_8$.

[6] In the latter case it is not clear to us if there is a consistent non-trivial theory; in the former case, the consistency of the theory in its "flat limit" will be discussed below.

JHEP02(2005)065

In the "flat limit" $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1) \to \mathbb{R}_\phi \times S^1$ (or $\mathbb{R}_\phi \times \mathbb{R}_t$ in the lorentzian case), where $\mathbb{R}_\phi$ is the SCFT of a scalar with a linear dilaton[7] (and $\mathbb{R}_t$ is time), the $\mathrm{U}(1) \times \mathrm{SO}(22)$ and $\mathrm{SU}(5) \times E_8$ are enhanced to $\mathrm{SO}(24)$ and $\mathrm{SO}(8) \times E_8$, respectively. The enhanced symmetry is due to the existence of two extra free fermions — ψ_ϕ and ψ_t — which together with the 22 λ's now give rise to these enlarged gauge groups. In this case, the one loop partition function of a 2-d heterotic string with a diagonal GSO projection was computed in [5]. For the SO(24) string it is the modular invariant function

$$Z_{\mathrm{SO}(24)}(\tau) = \frac{1}{2\eta^{12}} \left(\theta_3^{12} - \theta_4^{12} - \theta_2^{12}\right), \tag{5.6}$$

where $\theta_{2,3,4}$ are "theta functions" and η is the "Dedekind eta function" (see e.g. [24] for a review on such functions, and [5] and references therein for a detailed computation of this partition function). Actually, this modular invariant function is a constant $Z_{\mathrm{SO}(24)}(\tau) = 24$. Indeed, the physical spectrum of the theory consists in this case of 24 massless scalars in the fundamental representation of SO(24), in the (NS,NS) sector, whose corresponding vertex operators are[8]

$$V^I = e^{-\bar{\varphi}(\bar{z})} \lambda^I(z) e^{-\phi(z,\bar{z})}, \qquad I = 1, \ldots, 24, \tag{5.7}$$

where λ^I are the 24 free fermions constituting the SO(24) "internal space." For the $\mathrm{SO}(8) \times E_8$ theory one finds

$$Z_{\mathrm{SO}(8)\times E_8}(\tau) = \frac{1}{4\eta^{12}} \left(\theta_3^4 - \theta_4^4 - \theta_2^4\right) \left(\theta_3^8 + \theta_4^8 + \theta_2^8\right). \tag{5.8}$$

Here $Z_{\mathrm{SO}(8)\times E_8}(\tau) = 0$, and indeed there is an equal number of space-time bosons and fermions: 8 massless scalars in the fundamental representation 8_v of SO(8), in the (NS,NS) sector, whose vertex operators are

$$V^a = e^{-\bar{\varphi}(\bar{z})} \lambda^a(z) e^{-\phi(z,\bar{z})}, \qquad a = 1, \ldots, 8, \tag{5.9}$$

where λ^a are the 8 free worldsheet fermions constituting the SO(8) part of internal space, and 8 chiral and anti-chiral space-time fermions, in the (R,R) sector,[9] with vertex operators:

$$\chi_\alpha = e^{-\frac{\bar{\varphi}}{2}} e^{+\frac{i}{2}\bar{\mathcal{H}}} \mathcal{S}_\alpha e^{-\phi}, \qquad \bar{\chi}_{\bar{\alpha}} = e^{-\frac{\bar{\varphi}}{2}} e^{-\frac{i}{2}\bar{\mathcal{H}}} \mathcal{S}_{\bar{\alpha}} e^{-\phi}, \tag{5.10}$$

where $\mathcal{S}_\alpha$ and $\mathcal{S}_{\bar{\alpha}}$ are spin fields in the 8_s and 8_c spinor representations of SO(8), and $i\partial\bar{\mathcal{H}} = \bar{\psi}_t \bar{\psi}_\phi$.

[7] To obtain a weakly coupled string theory, we can re-add either an $N = 1$ Liouville superpotential or an $N = 2$ Liouville superpotential. The latter case is dual to the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ SCFT [32]. In the former case only a diagonal GSO projection can be done — the one discussed below.

[8] For simplicity, here and below we write the vertex operators corresponding to zero energy; for non-zero energy E an e^{iEt} piece should be added and the Liouville dressing factor should be changed accordingly to $e^{(-1+iE)\phi}$, so that the on-shell mass condition is satisfied.

[9] Note that in the heterotic string every operator whose right-moving part is in the Ramond sector corresponds to a space-time fermion.

JHEP02(2005)065

Remarkably, as was observed more than a year ago [3], the partition functions (5.6) and (5.8) are identical to the partition functions in the $N = (2,1)$ heterotic string, computed in [33] and as appear in eqs. (3.1) and (3.17) of [34], respectively. Moreover, the physical spectrum in (5.7) and (5.9), (5.10) is identical to the one in the $N = (2,1)$ heterotic string; there is a one to one correspondence with the vertex operators in eqs. (3.2) and (3.20), (3.21) of [34], respectively. There is however an important difference between the two theories: the $N = (2,1)$ heterotic string in flat space has a $1+1$ dimensional Poincaré symmetry, while here the space-like direction ϕ has a linear dilaton, with all its consequences. Nevertheless, it is natural to expect that the 2-d heterotic string is dual to the $N = (2,1)$ string on a different background.

We conjecture [3] that the 2-d heterotic string on $\mathbb{R}_\phi \times \mathbb{R}_t$ is dual to the $N = (2,1)$ string on $\mathbb{R}_\phi \times \mathbb{R}_t \times \mathbb{R}_x \times \mathbb{R}_\rho$, where $\mathbb{R}_\rho$ is the SCFT of a time-like scalar with a linear dilaton and $\mathbb{R}_x$ is a space-like scalar.[10] Fixing the null abelian gauge by setting $p_x = p_\rho = 0$, we obtain[11] in vertex operators a factor $e^{iEt+(-1+iE)\phi}e^{-\rho}$ (instead of the factor $e^{iEt+iEx}$ in flat space). Now the two theories have the same space-time symmetries. It remains to show that such an $N = (2,1)$ string is consistent and that its correlators are identical to those of the 2-d heterotic string.

We now turn to the two dimensional heterotic string on EBH $\times\, \mathcal{N}$. In this case the right-moving part of $\mathcal{N}$ is trivial, and hence the $\mathrm{SL}(2,\mathbb{R})$ level is set to $k = 1/2$, as above, such that the left-moving central charge is critical ($3 + 6/k = 15$). The left-moving bosonic sector consists of the SCFT on EBH (with $c = 15$) and an additional internal chiral sector which can be constructed, for instance, by the same 22 free chiral fermions as above. Two out of these fermions correspond now to the fermionization of x_L — the internal direction connected to the gauge field on EBH. The global gauge symmetry is now $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1) \times \mathrm{SO}(20)$ or $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1) \times \mathrm{SO}(4) \times E_8$. The former is obtained as follows: $\mathrm{U}(1) \times \mathrm{SO}(22)$ is broken by the gauge field to $\mathrm{U}(1)^2 \times \mathrm{SO}(20)$, and one of the U(1)'s is enhanced to an $\mathrm{SL}(2,\mathbb{R})$. The latter is a consequence of the breaking $\mathrm{SU}(5) \to \mathrm{U}(1)^2 \times \mathrm{SO}(4)$ by the EBH background, with the enhancement of one U(1) to an $\mathrm{SL}(2,\mathbb{R})$.[12] Since the $\mathrm{SL}(2,\mathbb{R})$ global symmetry is actually further enhanced to a full Virasoro algebra, the global space-time Lie symmetry $G = \mathrm{U}(1) \times \mathrm{SO}(20)$ or $\mathrm{U}(1) \times \mathrm{SO}(4) \times E_8$ is enhanced to affine G in the dual CFT_1, as in eq. (2.27) of [10].

In a similar way, one can construct two dimensional heterotic strings on the non-extremal 2-d CBH backgrounds (2.10). Two dimensional string theories in asymptotically flat space (with a linear dilaton) are expected to be dual to certain large-N Matrix Quantum Mechanics theories (MQM), which can be described by $N \to \infty$ free fermions in an inverted harmonic oscillator potential. It is thus natural to conjecture that for the 2-d heterotic string on the asymptotically flat 2-d non-extremal CBH background (2.10) a similar duality

[10] This SCFT on $\mathbb{R}^{2,2}$ with a null-like linear dilaton is the "near horizon" of a solitonic configuration in $\mathbb{R}^{2,2}$, of the sort studied in [35].

[11] The gauging in a direction with a linear dilaton may be done along the lines of eqs. (6.40)–(6.42) in [36]. Gauging the $\mathbb{R}^{1,1}_{t,x}$ directions we would obtain, instead, an $\mathbb{R}^{1,1}$ with a null-like linear dilaton.

[12] There are other ways to pick up a U(1) subgroup — with a level bigger than 1 — which will break SO(22) or SU(5) differently [37]; we thank D. Israel for pointing this out to us.

JHEP02(2005)065

holds (perhaps along the lines of the proposal in section 6 of [25] for the type 0 string theory on the uncharged 2-d black hole). Such MQM should have a global symmetry G.

For the extremal black hole we have obtained, instead, a two dimensional heterotic string on AdS_2. In this case we expect a dual CFT_1. Such a CFT_1 can perhaps be similar to the ones proposed for type 0A or type-IIA string theory on AdS_2: a certain Conformal Quantum Mechanics, which might be related to (a supersymmetric extension of) a theory of free fermions in a potential of the form $V \simeq Q^2/r^2$ (as proposed in [38] for type 0A on AdS_2 with Q units of RR flux), or perhaps a version of a supersymmetric Calogero-Moser model (as proposed for type-IIA on AdS_2 in [39, 40], where the above eigenvalue potential $V \simeq \sum_{i=1}^n \frac{Q^2}{r_i^2}$ is replaced by $V \simeq \sum_{i<j}^n \frac{Q^2}{(r_i - r_j)^2}$). Other possibilities involve the CQM duals associated with the near horizon physics of supersymmetric multi-black holes [41], any scale invariant one particle hamiltonians and in particular the hamiltonian of a free relativistic particle.

There is however a serious problem with these CFT_1 candidates. The first candidate [42], as well as any scale invariant one particle hamiltonian, has an $\mathrm{SL}(2,\mathbb{R})$ symmetry. However, although it is possible to enhance it to a Virasoro algebra classically [43], we do not know how to do it quantum mechanically. The Calogero models and the relativistic particle have a Virasoro and other affine symmetries [44, 43], but they (as for any quantum mechanics with a finite number of degrees of freedom) have zero central charges. On the other hand, our space-time theories have a Virasoro symmetry with central charge $c_{st} \simeq 1/g_s^2$ (see the next section) and affine symmetries with level $k_{st} \simeq 1/g_s^2$. Hence, it seems that the dual space-time theory has the properties of an *holomorphic* CFT_2 with a large central charge — closely related to the symmetric product duals to string theory on AdS_3, as follows from the facts in appendix B and the study in [30, 31] — rather than one of the known CFT_1's discussed above.

6. Entropy and reflection

In this section we discuss the entropy of the 2-d extremal black hole evaluated by using some aspects of its non-perturbative space-time dual, and the reflection coefficient of waves scattered from its event horizon by using string perturbation theory in the exact CFT description.

Some thermodynamical properties of the 2-d CBH (2.10) were studied semi-classically in [1, 9]. The entropy of the CBH is[13]

$$S_{\mathrm{CBH}} \simeq r_+ \,, \tag{6.1}$$

where r_+ is given in (2.13). In particular, in the extremal case the entropy is

$$S_{\mathrm{EBH}} \simeq M \simeq \frac{1}{g_s^2} \,, \tag{6.2}$$

where in the last equality we used the dependence of the 2-d black hole mass on the string coupling (2.12).

[13] We shall not follow (M, Q)-independent (though k-dependent) factors in the entropy.

In string theory on EBH $\times \mathcal{N}$ it is interesting to understand how to obtain such an entropy by counting the number of microstates. Generically, the study in this work provides a partial answer. Since the EBH background is equivalent to the extremal BTZ (see appendix B), they share the same entropy. In string theory on the BTZ black hole, the degrees of freedom consist of states in the Ramond sector with $L_0 + \bar{L}_0 \simeq M_{BTZ}$ and $L_0 - \bar{L}_0 \simeq J_{BTZ}$, in a dual space-time CFT$_2$ [45, 46, 10]. The semiclassical entropy of the BTZ black hole indeed matches the number of states in this CFT$_2$, as obtained from the Cardy formula [46]:

$$S_{BTZ} = 2\pi \left(\sqrt{\frac{c_{st} L_0}{6}} + \sqrt{\frac{c_{st} \bar{L}_0}{6}} \right), \tag{6.3}$$

when the central charge of the space-time CFT, c_{st}, is equal to the effective central charge (see also the discussion in [47, 10]). The KK reduction of the BTZ black hole to two dimensions yields a 2-d charged black hole [48, 2], whose near horizon behavior is identical to the the near horizon of our 2-d CBH (2.10), with $M_{BTZ} \simeq M$ and $J_{BTZ} \simeq Q$ (see eq. (2.9) in [2] and compare to (2.10)–(2.12), in the near horizon). Hence, the entropy in the extremal limit behaves like $\sqrt{c_{st} M}$,[14] and since $c_{st} \simeq 1/g_s^2$ [45, 46, 10], we obtain the same result as in eq. (6.2). We thus conclude that the same near horizon microstates constituting the entropy of the extremal BTZ black hole, lead to an entropy of the 2-d EBH which is linear in the black hole mass, as expected semiclassically.

Is eq. (6.2) valid also for the near horizon microstates in the two dimensional heterotic string on EBH? Since the extremal BTZ has $L_0 \simeq M$ and $\bar{L}_0 = 0$, it is plausible that the same space-time right-moving Ramond ground states with left-moving excitations — originated from (non-perturbative) string excitations and constituting the entropy of the BTZ black hole – also give rise to the same entropy $S \simeq M$ in the 2-d heterotic string on EBH. If true, this result should be also compatible with the number of states in the CFT$_1 \equiv CMQM$ dual to the 2-d heterotic string on EBH $\equiv AdS_2$. If, on the other hand, the 2-d heterotic string on EBH does not have the entropy of a black hole, then also a BTZ with level $k = 1/2$ does not have the Bekenstein-Hawking entropy in string theory.

The reflection coefficient $R(p)$ of a wave scattered from the event horizon of the 2-d CBH can be obtained in perturbative string theory from the two point function of the exact CFT background, by a formal analytic continuation, as in [49, 50, 8, 25]. For vertex operators of the type (4.1) one finds

$$R(j; m, \bar{m}) = \frac{\Gamma(1 - \frac{2j+1}{k})}{\Gamma(1 + \frac{2j+1}{k})} \frac{\Gamma(-2j-1)}{\Gamma(2j+1)} \frac{\Gamma(1+j+im)\Gamma(1+j+i\bar{m})}{\Gamma(-j+im)\Gamma(-j+i\bar{m})}, \tag{6.4}$$

where j is related to the momentum p along the radial direction by $j = -\frac{1}{2} + ip$ and $(m, \bar{m})$ are related to each other and to p by the gauge condition and the mass shell condition; hence m is related to the energy. As discussed in [8], one obtains $|R(p)| < 1$. For instance,

[14] Note that although the KK reduction of the BTZ black holes to 2-d yields asymptotically AdS_2 black holes [48, 2], unlike (2.10)–(2.12) which are asymptotically flat, we expect the entropy of both black holes to be dominated by near horizon microstates, and hence to be similar even in the non-extremal case.

JHEP02(2005)065

for uncharged waves in the extremal case, using the gauge condition (4.2) in (6.4), one finds

$$|R(p,m)| = \frac{\cosh(\pi(2p-m)) + \cosh(\pi m)}{\cosh(\pi(2p+m)) + \cosh(\pi m)}\,. \tag{6.5}$$

For incoming waves with energy m and momentum p ($p, m > 0$), we thus obtain

$$|R(p)| < 1\,. \tag{6.6}$$

Therefore, part of the incoming flux is absorbed by the black hole.

Acknowledgments

The relation of the 2-d heterotic string to the $N = (2,1)$ string, and the conjecture that they are dual, was done with David Kutasov [3] during A.G.'s visit at the EFI and the Department of Physics at the University of Chicago in September, 2003. We thank O. Aharony, M. Berkooz, D. Israel and D. Kutasov for discussions. A.G. and A.S. thank the EFI and the Department of Physics at the University of Chicago for its warm hospitality. A.G. thanks the MPI for Physics in Munich for its warm hospitality. This work was supported in part by the Israel Academy of Sciences and Humanities — Centers of Excellence Program, by GIF — German-Israel Bi-National Science Foundation, and the European RTN network HPRN-CT-2000-00122. The work of A.S. is supported in part by the Horowitz Foundation.

A. AdS_2

Two dimensional Anti-de-Sitter space (AdS_2) can be represented as (the infinite cover of) hyperboloid (see [22] for a review)

$$X_0^2 + X_2^2 - X_1^2 = R^2\,, \tag{A.1}$$

in $\mathbb{R}^{2,1}$ with metric

$$ds^2 = -dX_0^2 - dX_2^2 + dX_1^2\,. \tag{A.2}$$

There are various useful coordinate systems which we describe in the following subsections.

A.1 Global coordinates

A solution of (A.1) is given, for instance, by parametrizing

$$\begin{aligned} X_0 &= R\cosh\rho\cos\tau\,, \\ X_1 &= R\sinh\rho\,, \\ X_2 &= R\cosh\rho\sin\tau\,. \end{aligned} \tag{A.3}$$

Using (A.2) with (A.3), one obtaines the metric of AdS_2 as

$$ds^2 = R^2(-\cosh^2\rho d\tau^2 + d\rho^2)\,. \tag{A.4}$$

For $\rho \in \mathbb{R}$ and $0 \le \tau \le 2\pi$ the solution (A.3) covers the whole hyperboloid (A.1) exactly once. Hence, (τ, ρ) are called global coordinates of AdS_2. Near $\rho = 0$ the metric behaves as $ds^2 \to R^2(-d\tau^2 + d\rho^2)$, hence the hyperboloid has the topology of $S^1 \times \mathbb{R}$, where S^1 represents closed time-like curves in the τ direction. By unwraping τ to $-\infty < \tau < \infty$ we obtain the infinite cover of the hyperboloid, and avoid the problems of closed time-like curves. Usually, by AdS_2 we mean this universal cover.

A.2 The strip

To investigate the causal structure it is convenient to do a conformal transformation to flat space. For that purpose we introduce a coordinate φ, related to ρ by

$$\tan\varphi = \sinh\rho\,, \qquad -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}\,. \tag{A.5}$$

Now the metric (A.4) turns into

$$ds^2 = \frac{R^2}{\cos^2\varphi}(-d\tau^2 + d\varphi^2)\,, \tag{A.6}$$

and by a conformal transformation we thus obtain

$$ds'^2 = \frac{\cos^2\varphi}{R^2}ds^2 = -d\tau^2 + d\varphi^2\,. \tag{A.7}$$

This is a two dimensional flat strip, where time runs all along the strip, the two boundaries of AdS_2 are mapped to the boundaries of the strip at $\varphi = \pm\pi/2$, and light-like directions are 45 degrees lines in the (τ, φ) plain.

A.3 Poincaré coordinates

Parametrizing

$$\begin{aligned} X_0 &= \frac{1}{2u}\left(1 + u^2(R^2 - t^2)\right), \\ X_1 &= \frac{1}{2u}\left(1 - u^2(R^2 + t^2)\right), \\ X_2 &= \mathrm{Rut}\,, \qquad u \geq 0\,, \end{aligned} \tag{A.8}$$

one obtains the metric

$$ds^2 = R^2\left(-u^2dt^2 + \frac{du^2}{u^2}\right). \tag{A.9}$$

The coordinates (A.8) cover one half of the hyperboloid (A.1) — the Poincaré patch of AdS_2. They are thus called the Poincaré coordinates.

A.4 Black hole coordinates

Consider the metric

$$ds^2 = R^2\left(-(r^2 - m)d\tau^2 + \frac{dr^2}{r^2 - m}\right). \tag{A.10}$$

We shall refer to this background as an "AdS_2 black hole" with a mass related to m. Actually, by rescaling r and t we see that only the sign of m is important, so w.l.g. we consider only $m = \pm 1$. For $m = -1$, we find that the background

$$ds^2 = R^2\left(-(r^2 + 1)d\tau^2 + \frac{dr^2}{r^2 + 1}\right), \tag{A.11}$$

JHEP02(2005)065

is equivalent to AdS_2 in global coordinates (A.4) by the coordinate transformation

$$r = \sinh \rho \,. \tag{A.12}$$

On the other hand, for $m = 1$, the background (A.10) is

$$ds^2 = R^2 \left(-(r^2-1) d\tau^2 + \frac{dr^2}{r^2-1} \right) , \tag{A.13}$$

which is identical for $r \geq 1$ to the metric in (3.3), as can be seen by the coordinate transformation

$$r = \cosh \chi \,, \qquad \tau = y \,. \tag{A.14}$$

Hence, for $r \geq 1$ the background (A.13) covers region A in the extremal AdS_2 black hole. Similarly, $-1 \leq r \leq 1$ ($r = \cos \chi$, for which (A.13) is equal to (3.4)) covers region B, and $r \leq -1$ ($r = -\cosh \chi$) covers region C. The gauge field of (3.3), (3.4) in these r, τ coordinates is

$$A_\tau = \frac{1}{2}(1-r) \,. \tag{A.15}$$

Finally, the maximal extension of this 2-d black hole covers all of AdS_2 – the infinite cover of the hyperboloid.

B. EBH $\equiv AdS_3/\Gamma_R \equiv$ extremal BTZ

In this appendix we show that the CFT corresponding to the extremal 2-d black hole is equivalent to a certain (right-moving) orbifold of $AdS_3 \equiv \tilde{SL}(2,\mathbb{R})$, studied in [2, 19]. Before KK reduction, the metric in region A is

$$\frac{1}{k} ds^2 = d\theta^2 - \sinh^2 \theta dy^2 + 2 \sinh^2 \theta dy dx + dx^2 \,, \tag{B.1}$$

where x is compact and y, θ are the same as in section 2. After changing coordinates to

$$y = \theta_L \,, \qquad x = \frac{1}{2}(\theta_L + \theta_R) \,, \qquad \theta = \frac{\chi}{2} \,, \tag{B.2}$$

we find

$$\frac{4}{k} ds^2 = d\chi^2 + d\theta_L^2 + d\theta_R^2 + 2 \cosh \chi d\theta_L d\theta_R \,, \tag{B.3}$$

where θ_R is periodic (since x is compact). This background corresponds to region A of the orbifold of $\mathrm{SL}(2,\mathbb{R})$ by

$$g \simeq g e^{\pi r_R \sigma_3} \,, \tag{B.4}$$

where r_R is a constant. This can be seen by noting that the background (B.1) is obtained from a WZW model on $\mathrm{SL}(2,\mathbb{R})$ with $g \in \mathrm{SL}(2,\mathbb{R})$ parametrized as in (2.2) by

$$g = e^{\theta_L \sigma_3/2} g(\chi) e^{\theta_R \sigma_3/2} \,, \tag{B.5}$$

where $g(\chi) = e^{\chi \sigma_1/2}$ in region A, as in (2.3). Hence, the exact fully asymmetric $\frac{\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)}{\mathrm{U}(1)}$ quotient CFT — leading to the EBH (or AdS_2 with gauge field) upon KK reduction – is equivalent to an orbifold of AdS_3.

Finally, let us note that this background is actually equivalent to the extremal BTZ black hole [51]. Such three dimensional black holes, with mass $M_{BTZ} = r_+^2 + r_-^2$ and angular momentum $J_{BTZ} = 2r_+r_-$, can be constructed as orbifolds of the SL$(2,\mathbb{R})$ CFT [52, 53] by (see e.g. [54] for similar notation to those we use):

$$g \simeq e^{\pi(r_+ - r_-)\sigma_3} g e^{\pi(r_+ + r_-)\sigma_3} . \tag{B.6}$$

Hence, the orbifold (B.4) is the BTZ black hole with $M_{BTZ} = |J_{BTZ}|$.

References

[1] C.R. Nappi and A. Pasquinucci, *Thermodynamics of two-dimensional black holes*, *Mod. Phys. Lett.* **A 7** (1992) 3337 [gr-qc/9208002].

[2] D.A. Lowe and A. Strominger, *Exact four-dimensional dyonic black holes and Bertotti-Robinson space-times in string theory*, *Phys. Rev. Lett.* **73** (1994) 1468 [hep-th/9403186].

[3] A. Giveon and D. Kutasov, unpublished.

[4] A. Giveon, M. Porrati and E. Rabinovici, *Target space duality in string theory*, *Phys. Rept.* **244** (1994) 77 [hep-th/9401139].

[5] M.D. McGuigan, C.R. Nappi and S.A. Yost, *Charged black holes in two-dimensional string theory*, *Nucl. Phys.* **B 375** (1992) 421 [hep-th/9111038].

[6] A. Giveon and M. Roček, *Generalized duality in curved string backgrounds*, *Nucl. Phys.* **B 380** (1992) 128 [hep-th/9112070].

[7] C.V. Johnson, *Exact models of extremal dyonic 4-D black hole solutions of heterotic string theory*, *Phys. Rev.* **D 50** (1994) 4032 [hep-th/9403192].

[8] A. Giveon, E. Rabinovici and A. Sever, *Beyond the singularity of the 2-D charged black hole*, *JHEP* **07** (2003) 055 [hep-th/0305140].

[9] A. Giveon, A. Konechny, E. Rabinovici and A. Sever, *On thermodynamical properties of some coset CFT backgrounds*, *JHEP* **07** (2004) 076 [hep-th/0406131].

[10] A. Giveon, D. Kutasov and N. Seiberg, *Comments on string theory on AdS_3*, *Adv. Theor. Math. Phys.* **2** (1998) 733 [hep-th/9806194].

[11] A. Giveon, *Target space duality and stringy black holes*, *Mod. Phys. Lett.* **A 6** (1991) 2843.

[12] A. Giveon, B. Kol, A. Ori and A. Sever, *On the resolution of the time-like singularities in Reissner-Nordstroem and negative-mass schwarzschild*, *JHEP* **08** (2004) 014 [hep-th/0401209].

[13] A.A. Tseytlin, *Conformal sigma models corresponding to gauged Wess-Zumino-Witten theories*, *Nucl. Phys.* **B 411** (1994) 509 [hep-th/9302083].

[14] A. Giveon, E. Rabinovici and A.A. Tseytlin, *Heterotic string solutions and coset conformal field theories*, *Nucl. Phys.* **B 409** (1993) 339 [hep-th/9304155].

[15] I. Jack, D.R.T. Jones and J. Panvel, *Exact bosonic and supersymmetric string black hole solutions*, *Nucl. Phys.* **B 393** (1993) 95 [hep-th/9201039].

[16] I. Bars and K. Sfetsos, *Conformally exact metric and dilaton in string theory on curved space-time*, *Phys. Rev.* **D 46** (1992) 4510 [hep-th/9206006].

[17] K. Sfetsos, *Conformally exact results for* $SL(2,\mathbb{R}) \times SO(1,1)(d-2)/SO(1,1)$ *coset models*, *Nucl. Phys.* **B 389** (1993) 424 [hep-th/9206048].

[18] I. Bars and K. Sfetsos, $SL(2-R) \times SU(2)/IR^2$ *string model in curved space-time and exact conformal results*, *Phys. Lett.* **B 301** (1993) 183 [hep-th/9208001].

[19] A. Strominger, *AdS_2 quantum gravity and string theory*, *JHEP* **01** (1999) 007 [hep-th/9809027].

[20] D. Israel, C. Kounnas, D. Orlando and P.M. Petropoulos, *Electric/magnetic deformations of S^3 and AdS_3 and geometric cosets*, *Fortschr. Phys.* **53** (2005) 73 [hep-th/0405213].

[21] A. Giveon and E. Kiritsis, *Axial vector duality as a gauge symmetry and topology change in string theory*, *Nucl. Phys.* **B 411** (1994) 487 [hep-th/9303016].

[22] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[23] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory. Vol. 1: Introduction; Vol. 2: Loop amplitudes, anomalies and phenomenology*, Cambridge University Press, Cambridge 1987.

[24] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string; Vol. 2: Superstring theory and beyond*, Cambridge University Press, Cambridge 1998.

[25] A. Giveon, A. Konechny, A. Pakman and A. Sever, *Type-0 strings in a 2-D black hole*, *JHEP* **10** (2003) 025 [hep-th/0309056].

[26] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[27] A. Giveon and M. Roček, *Supersymmetric string vacua on $AdS_3 \times N$*, *JHEP* **04** (1999) 019 [hep-th/9904024].

[28] D. Berenstein and R.G. Leigh, *Spacetime supersymmetry in AdS_3 backgrounds*, *Phys. Lett.* **B 458** (1999) 297 [hep-th/9904040].

[29] A. Giveon and A. Pakman, *More on superstrings in $AdS_3 \times N$*, *JHEP* **03** (2003) 056 [hep-th/0302217].

[30] R. Argurio, A. Giveon and A. Shomer, *Superstrings on AdS_3 and symmetric products*, *JHEP* **12** (2000) 003 [hep-th/0009242].

[31] R. Argurio, A. Giveon and A. Shomer, *String theory on AdS_3 and symmetric products*, *Fortschr. Phys.* **49** (2001) 409 [hep-th/0012117].

[32] A. Giveon and D. Kutasov, *Little string theory in a double scaling limit*, *JHEP* **10** (1999) 034 [hep-th/9909110].

[33] D.M. Pierce, *A (1,2) heterotic string with gauge symmetry*, *Phys. Rev.* **D 53** (1996) 7197 [hep-th/9601125].

[34] D. Kutasov and E.J. Martinec, *New principles for string/membrane unification*, *Nucl. Phys.* **B 477** (1996) 652 [hep-th/9602049].

JHEP02(2005)065

[35] M.J. O'Loughlin and S. Randjbar-Daemi, *$AdS_3 \times R$ as a target space for the (2,1) string theory*, *Nucl. Phys.* **B 543** (1999) 170 [hep-th/9807208].

[36] S. Elitzur, A. Giveon, E. Rabinovici, A. Schwimmer and G. Veneziano, *Remarks on nonabelian duality*, *Nucl. Phys.* **B 435** (1995) 147 [hep-th/9409011].

[37] D. Israel, C. Kounnas, D. Orlando and P.M. Petropoulos, *Heterotic strings on homogeneous spaces*, hep-th/0412220.

[38] A. Strominger, *A matrix model for AdS_2*, *JHEP* **03** (2004) 066 [hep-th/0312194].

[39] G.W. Gibbons and P.K. Townsend, *Black holes and calogero models*, *Phys. Lett.* **B 454** (1999) 187 [hep-th/9812034].

[40] H. Verlinde, *Superstrings on AdS_2 and superconformal matrix quantum mechanics*, hep-th/0403024.

[41] J. Michelson and A. Strominger, *Superconformal multi-black hole quantum mechanics*, *JHEP* **09** (1999) 005 [hep-th/9908044]; *The geometry of (super)conformal quantum mechanics*, *Commun. Math. Phys.* **213** (2000) 1 [hep-th/9907191];
A. Maloney, M. Spradlin and A. Strominger, *Superconformal multi-black hole moduli spaces in four dimensions*, *JHEP* **04** (2002) 003 [hep-th/9911001];
R. Britto-Pacumio, A. Strominger and A. Volovich, *Two-black-hole bound states*, *JHEP* **03** (2001) 050 [hep-th/0004017].

[42] V. de Alfaro, S. Fubini and G. Furlan, *Conformal invariance in quantum mechanics*, *Nuovo Cim.* **A34** (1976) 569.

[43] J. Kumar, *Conformal mechanics and the virasoro algebra*, *JHEP* **04** (1999) 006 [hep-th/9901139].

[44] E. Bergshoeff and M.A. Vasiliev, *The calogero model and the virasoro symmetry*, *Int. J. Mod. Phys.* **A 10** (1995) 3477 [hep-th/9411093].

[45] J.D. Brown and M. Henneaux, *Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity*, *Commun. Math. Phys.* **104** (1986) 207.

[46] A. Strominger, *Black hole entropy from near-horizon microstates*, *JHEP* **02** (1998) 009 [hep-th/9712251].

[47] S. Carlip, *What we don't know about BTZ black hole entropy*, *Class. and Quant. Grav.* **15** (1998) 3609 [hep-th/9806026].

[48] A. Achucarro and M.E. Ortiz, *Relating black holes in two-dimensions and three-dimensions*, *Phys. Rev.* **D 48** (1993) 3600 [hep-th/9304068].

[49] R. Dijkgraaf, H. Verlinde and E. Verlinde, *String propagation in a black hole geometry*, *Nucl. Phys.* **B 371** (1992) 269.

[50] S. Elitzur, A. Giveon, D. Kutasov and E. Rabinovici, *From big bang to big crunch and beyond*, *JHEP* **06** (2002) 017 [hep-th/0204189].

[51] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [hep-th/9204099].

[52] G.T. Horowitz and D.L. Welch, *Exact three-dimensional black holes in string theory*, *Phys. Rev. Lett.* **71** (1993) 328 [hep-th/9302126].

[53] N. Kaloper, *Miens of the three-dimensional black hole*, *Phys. Rev.* **D 48** (1993) 2598 [hep-th/9303007].

[54] S. Hemming and E. Keski-Vakkuri, *The spectrum of strings on btz black holes and spectral flow in the* $\mathrm{SL}(2,\mathbb{R})$ *WZW model*, *Nucl. Phys.* **B 626** (2002) 363 [hep-th/0110252].